A Relationship between N-square diagrams and K-maps
By Michelle Thompson, 2 March 2006

A Karnaugh map (K-map) provides a pictorial method of grouping together binary expressions with common factors and therefore eliminating unwanted variables. The K-map can also be described as a special arrangement of a truth table.

An N-square diagram is a method of presenting the relationships between interfaces. It also can be described as a special arrangement of a truth table.

This paper examines the structure of both the K-map and the N-square diagrams with the goal of showing a relationship.

Taking an N-square diagram,

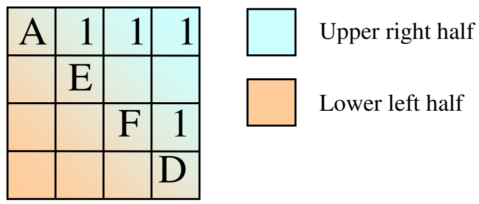
A			
	E		
		F	
			D

A = Airframe
E = Engine
F = Fuel
D = DHL

The interfaces are then marked with the number 1 if an interface exists between the row and column that the "1" occurs in. If there is no interface between the system elements, then that square is left blank. For example, there are interfaces between A and E, F, and D and between F and D.

A	1	1	1
	E		
		F	1
			D

Because the N-square map is directional (interface relationships are clockwise) we take one half at a time. Halves are divided by the diagonal interface elements. Therefore the upper right and the lower left are the two halves of the N-square diagram.



The reason for the division is because in a K-map each square is a unique term while in an N-square diagram the interface between A and E can be defined in two ways. First, clockwise from A to E, then clockwise again from E to A. This means that the directionality of the N-square diagram increases the apparent dimensionality. AE ≠ EA. There can be an interface from A to E while no interface exists from E to A. There are two different terms produced. However, on a K-map, AE = EA (i.e. AE = EA = 11). It’s the same term. For a comparison between N-square diagrams and K-maps, this difference is important.

Taking the diagonal elements of the N-square map and setting them as the variables in a K-map reveals the relationship.

> <u>Some helpful definitions for the next step</u>
>
> A *product term* is a logic-AND of binary variables, where each variable is either a variable or its negation. For example, take the binary function *f*(x,y,z) = xz + x’y the two product terms are xz and x’y.
>
> A product term is an implicant of a binary function if the product term defines the function.
>
> A prime implicant is a minimal implicant. Nothing can be removed from the term without wrecking the function. There are no redundant terms. K-maps are used to eliminate redundant terms.
>
> The sum of all prime implicants of a binary function is called the *complete sum* of that function.

Interestingly, each square in the N-square diagram corresponds to a *prime implicant* in a corresponding K-map.

N-square diagrams

corresponding K-maps

A	1		
	E		
		F	
			D

A interfaced to E is represented by setting A and E both equal to one. Anywhere on the K-map where A and E are 1 is marked with a 1.

AE \ FD	00	01	11	10
00				
01				
11	1	1	1	1
10				

A		1	
	E		
		F	
			D

A interfaced to F is represented by setting A and F both equal to one. Anywhere on the K-map where A and F are 1 is marked with a 1.

AE \ FD	00	01	11	10
00				
01				
11			1	1
10			1	1

A			1
	E		
		F	
			D

A interfaced to D is represented by setting A and D both equal to one. Anywhere on the K-map where A and D are 1 is marked with a 1.

AE \ FD	00	01	11	10
00				
01				
11		1	1	
10		1	1	

N-square diagrams

corresponding K-maps

A			
	E	1	
		F	
			D

E interfaced to F is represented by setting E and F both equal to one. Anywhere on the K-map where E and F are 1 is marked with a 1.

AE \ FD	00	01	11	10
00				
01			1	1
11			1	1
10				

A			
	E		1
		F	
			D

E interfaced to D is represented by setting E and D both equal to one. Anywhere on the K-map where E and D are 1 is marked with a 1.

AE \ FD	00	01	11	10
00				
01		1	1	
11		1	1	
10				

A			
	E		
		F	1
			D

F interfaced to D is represented by setting F and D both equal to one. Anywhere on the K-map where F and D are 1 is marked with a 1.

AE \ FD	00	01	11	10
00			1	
01			1	
11			1	
10			1	

A	1	1	1
	E	1	1
		F	1
			D

Let's set all the interfaces connecting together in the upper right half of the N-square diagram. What about the squares left over?

AE \ FD	00	01	11	10
00			1	
01		1	1	1
11	1	1	1	1
10		1	1	1

Each of the leftover squares corresponds to a particular term of the K-map. Making a table of those leftover squares, a pattern emerges.

A	E	F	D
0	0	0	0
0	0	0	1
0	0	1	0
0	1	0	0
1	0	0	0

Following from the definitions that we established for the translation of N-square to K-map, the first term would be if there were no interfaces between system elements on the N-square diagram at all. The second through fifth terms are the elements of the diagonal themselves. In other words, it is shown that D interfaces with D, F interfaces with F, E interfaces with E, and A interfaces with A. These *self-identity* terms along with the case of the all-zeroes "no interfaces at all" condition complete the K-map.

Therefore, the upper right half of the K-square diagram corresponds to a complete K-map. Each individual interface between each system element corresponds to a prime implicant on the K-map. Therefore, each half of an N-square diagram is already irreducible.

This means that an N-square diagram is already in simplest form, and methods associated with K-maps will not reduce the number of interfaces further.

The dimensionality of the above N-square diagram is 4. There are 4 elements along the diagonal. The dimensionality of the K-map for each half is also 4. Since two K-maps are required, then the equivalent dimensionality of the corresponding K-map would appear to be 2*N (N is the dimension of the N-diagram), as each pairing of system elements creates two different combinations ($AE \neq EA$).

Since order matters, the K-map variables could be expressed as A E F D A' E' F' D'. There is overlap between the two K-maps since each K-map shares the diagonal elements when they interface only with themselves. These *self-identity* terms can be considered equivalent whether they were derived from the upper right half or the lower left half.

This reduces the dimensionality of the equivalent K-map by at least the number of diagonal elements.

The case of there being no interfaces on the upper right half and the case of there being no interfaces on the lower left half are two different cases, so they are not combined like the *self-identity* cases.

Therefore, the dimensionality of the corresponding K-map is [(2*N) – N] or N. This means that the dimensionality of an N-diagram and a K-map are equivalent.